PR, then inequality (15) will be satisfied, and the perturbed residual eigenvalues can never be brought into the right-half plane.

For the case where the $\mathrm{Re}(\lambda_k)$ are not sufficiently small but satisfying $\max\{\mathrm{Re}[\lambda(A_g)\cup\lambda(A_e)]\} < \min[\mathrm{Re}(\lambda_k)], k = n+1, n+2, \ldots,$ we can change the complex variable s to be $s = p + \pi$, where $p \in C, \pi = \max\{\mathrm{Re}[\lambda(A_g) \cup \lambda(A_e)]\}$, and express $\Pi(s)$ as

$$\Pi(s) = \Pi(p + \pi) = \tilde{\Pi}(p)$$

Since $\tilde{\Pi}(p)$ is analytic for all $\mathrm{Re}(p) > 0$, using Definition P4, we see that the shift of Eq. (12) will be nonpositive if $\tilde{\Pi}(p)$ is PR, i.e.,

$$\tilde{\Pi}(p) + \tilde{\Pi}^*(p) \geq 0 \qquad \forall \mathrm{Re}(p) > 0$$

For a flexible-structure system described in the form of transfer matrices (i.e., I/O description), it is well known that if a controlled flexible system is PR (respectively, SPR), then an SPR (respectively, PR) compensator will stabilize the closed-loop system despite the existence of spillover.[1] Suppose the controlled flexible system is PR. We now show that an SPR compensator implies the strictly positive realness of $\Pi(s)$. First note that the compensator transfer matrix of Eq. (4) is given by

$$\phi_c(s) = G(sI_n - A_c + B_cG + KC_c)^{-1}K$$

and the controlled dynamics is

$$\phi_p(s) = C_c(sI_n - A_c)^{-1}B_c$$

By the definition of $\Pi(s)$ in Eq. (6), we obtain, successively,

$$\begin{aligned}
\Pi(s) &\\
&= G(sI_n - A_c + B_cG)^{-1}(sI_n - A_c)(sI_n - A_c + KC_c)^{-1}K \\
&= G\left\{sI_n - A_c + B_cG + KC_c + KC_c(sI_n - A_c)^{-1}B_cG\right\}^{-1}K \\
&= G(sI_n - A_c + B_cG + KC_c)^{-1}\left\{I_n - K\left[I_n + C_c(sI_n - A_c)^{-1}\right.\right. \\
&\quad \times B_cG(sI_n - A_c + B_cG + KC_c)^{-1}K\left.\right]^{-1}C_c(sI_n - A_c)^{-1} \\
&\quad \times B_cG(sI_n - A_c + B_cG + KC_c)^{-1}\left.\right\}K \\
&= G(sI_n - A_c + B_cG + KC_c)^{-1}K\left\{I_n - \left[I_n + C_c(sI_n - A_c)^{-1}\right.\right. \\
&\quad \times B_cG(sI_n - A_c + B_cG + KC_c)^{-1}K\left.\right]^{-1} \\
&\quad \times C_c(sI_n - A_c)^{-1}B_cG(sI_n - A_c + B_cG + KC_c)^{-1}K\left.\right\} \\
&= \left\{\left[G(sI_n - A_c + B_cG + KC_c)^{-1}K\right]^{-1}\right. \\
&\quad \left. + C_c(sI_n - A_c)^{-1}B_c\right\}^{-1} \qquad (16)
\end{aligned}$$

The matrix inversion formula has been used in the third step above. From Lemma 1, we know that an SPR transfer matrix $\phi_c(s)$ implies the strictly positive realness of $\phi_c^{-1}(s)$. Since $\phi_p(s)$ is assumed to be PR, based on the basic properties of the parallel combination of the PR blocks $\phi_c^{-1}(s)$ and $\phi_p(s)$, we may then conclude from Lemma 1 and Eq. (16) that $\Pi(s)$ is SPR. This implies the shifts of $\mathrm{Re}(\sigma_k)$, $k = n + 1, n + 2, \ldots,$ in Eq. (14) would not be positive. Also note that if $\phi_p(s)$ is SPR, then a PR transfer matrix $\phi_c(s)$ implies the strictly positive realness of $\Pi(s)$ and the shifts of residual poles could still be nonpositive.

Conclusions

In this Note, we have investigated residual pole shifts of a class of flexible structures under low-order observer-based control. Permissible magnitudes of residual pole shifts that ensure residual poles retained in the open left-half plant given that the closed-loop is stable have been presented. It was shown that the derived results are consistent with the positive real control approaches.

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Regulation of Single-Link Flexible Manipulator Involving Large Elastic Deflections

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I. Introduction

THE control of single-link flexible manipulators has been widely studied in the literature. A vast majority of these studies (too numerious to mention) have designed controllers based on truncated finite-dimensional models. Only in a few related papers[1–7] has there been construction of controllers making direct use of distributed parameter models. However, the small-deflection assumption has been invoked in nearly all these studies (using either truncated finite-dimensional models or infinite-dimensional models).

In this Note, the rest-to-rest maneuver of a horizontally slew torque-driven beam undergoing geometrically exact elastic deflections is considered. The equations of motion for a hub–beam system are derived first. Instead of the approach[8,9] employing Hamilton's principle, this derivation uses Newton's second law for the sake of retaining the simple physical structure of the problem. Then a simple linear feedback law using only the joint torque is obtained via a Lyapunov-type method. In deriving the feedback law, neither model truncation nor small deflection assumption is imposed. A proof of globally asymptotic stability of the closed-loop system is presented. A similar problem was considered in Ref. 9, but no formal proof of asymptotic stability was provided.

We emphasize that 1) to admit large elastic deflections removes the speed and acceleration limitations imposed by the small-deflection assumption, 2) to adopt the distributed parameter modeling eliminates the truncation error caused by model simplification, and 3) to employ the geometrically exact description implies that no error in kinematics of elastic deformation is involved.

II. Equations of Motion

The horizontal slewing beam in the undeformed state depicted in Fig. 1 of length L, area moment of inertia I, cross-sectional area

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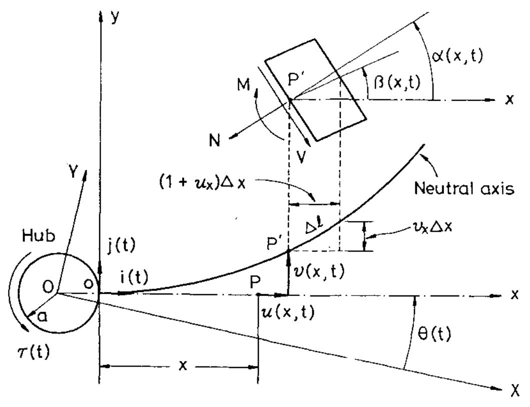

Fig. 1 Schematic of hub–beam system.

A, mass per unit length A_ρ, mass moment of inertia per unit length I_ρ, shear coefficient k_s, shear modulus G, and Young's modulus E is attached at one end to a rigid hub of radius a and mass moment of inertia I_h, which in turn is connected to an actuator that supplies a torque τ. Assume that the beam possesses a line of symmetry in the plane of bending, plane sections orthogonal to the neutral axis remain plane during deformation, and the line of centroids coincides with the neutral axis.

Let $o(x, y)$ be a floating frame fixed on the hub with x axis coincident with the neutral axis of the undeformed beam and $O(X, Y)$ be a fixed frame. The position vector of the point P' on the deformed neutral axis is given by

$$\boldsymbol{R}_{P'} = (a + x + u)\boldsymbol{i} + v\boldsymbol{j} \tag{1}$$

where $u(x, t)$ and $v(x, t)$ are axial and transverse displacements of the point P on the undeformed neutral axis and $\boldsymbol{i}(t)$ and $\boldsymbol{j}(t)$ are unit base vectors of the floating frame. The velocity and acceleration of the particle P' relative to the fixed frame, respectively, can be written as

$$\boldsymbol{V}_{P'} = (\dot{u} - v\dot{\theta})\boldsymbol{i} + [\dot{v} + (a + x + u)\dot{\theta}]\boldsymbol{j} \tag{2}$$

$$\boldsymbol{A}_{P'} = [\ddot{u} - 2\dot{v}\dot{\theta} - v\ddot{\theta} - (a + x + u)\dot{\theta}^2]\boldsymbol{i} + [\ddot{v} + 2\dot{u}\dot{\theta} + (a + x + u)\ddot{\theta} - v\dot{\theta}^2]\boldsymbol{j} \tag{3}$$

The moment of inertia force of the elastic beam relative to the point O is

$$\begin{aligned} M_0 &= -\int_0^L A_\rho |\boldsymbol{R}_{P'} \times \boldsymbol{A}_{P'}|\,\mathrm{d}x - \int_0^L I_\rho \ddot{\alpha}\,\mathrm{d}x \\ &= -\int_0^L A_\rho\{[(a + x + u)^2 + v^2]\ddot{\theta} + 2[(a + x + u)\dot{u} + v\dot{v}]\dot{\theta} \\ &\quad + [(a + x + u)]\ddot{v} - \ddot{u}v]\}\,\mathrm{d}x - \int_0^L I_\rho \ddot{\alpha}\,\mathrm{d}x \end{aligned} \tag{4}$$

where $\alpha(x, t)$ is the angle of rotation of a beam cross section from its undeformed configuration and $a \ll L$ is assumed.

The rotational equation of motion of the hub–beam system can now be written as

$$\frac{\mathrm{d}}{\mathrm{d}t}\left[\left(\int_0^L \left\{A_\rho[(a + x + u)^2 + v^2] + I_\rho\right\}\mathrm{d}x\right)\dot{\theta} + \int_0^L A_\rho[(a + x + u)\dot{v} - \dot{u}v]\,\mathrm{d}x + \int_0^L I_\rho \dot{\alpha}\,\mathrm{d}x\right] = \tau - I_h\ddot{\theta} \tag{5}$$

Note that Eq. (5) depends neither on the strains (small or large) nor on the stresses (constitutive equation) of the deformed beam. Note also that the left-hand side of Eq. (5) can be expressed as $I_b\ddot{\theta} + \dot{H}_f$, where $I_b = \int_0^L A_\rho(a + x)^2\,\mathrm{d}x$ is the mass moment of inertia of the undeformed beam and H_f (its expression is self-evident) corresponds to the angular momentum of the beam associated only with deformational motions.

In order to establish the equations of motion for the elastic deflections, a differential element of the beam is considered. One can easily show from $|\mathrm{d}\boldsymbol{R}_{P'}| = \mathrm{d}\ell$ that

$$(\mathrm{d}\ell)^2 = \left[(1 + u_x)^2 + v_x^2\right](\mathrm{d}x)^2 \tag{6}$$

$$\sin\beta = v_x\left[(1 + u_x)^2 + v_x^2\right]^{-\frac{1}{2}} \tag{7}$$

$$\cos\beta = (1 + u_x)\left[(1 + u_x)^2 + v_x^2\right]^{-\frac{1}{2}} \tag{8}$$

where $\beta(x, t)$ is the angle between the tangents of deformed and undeformed neutral axes and the subscript x denotes a partial derivative with respect to x. Let the longitudinal force, the shear force, and the bending moment acting on the element be denoted by N, V, and M, respectively. Summing the forces in x and y directions and the moments of force acting on the element, one obtains

$$A_\rho[\ddot{u} - v\ddot{\theta} - 2\dot{\theta}\dot{v} - \dot{\theta}^2(a + x + u)] = (N\cos\alpha)_x - (V\sin\alpha)_x \tag{9}$$

$$A_\rho[\ddot{v} + \ddot{\theta}(a + x + u) + 2\dot{\theta}\dot{u} - \dot{\theta}^2 v] = (N\sin\alpha)_x + (V\cos\alpha)_x \tag{10}$$

$$I_\rho(\ddot{\theta} + \ddot{\alpha}) = M_x - [N\sin(\beta - \alpha) - V\cos(\beta - \alpha)]\frac{\mathrm{d}\ell}{\mathrm{d}x} \tag{11}$$

Note that Eqs. (9–11) are independent of the constitutive equation of the beam.

To complete the formulation, the stress–strain relations for the homogeneous, isotropic linearly elastic beam can be taken as[10]

$$N = EA\left[\frac{\mathrm{d}\ell}{\mathrm{d}x}\cos(\beta - \alpha) - 1\right] \tag{12}$$

$$V = k_s GA\frac{\mathrm{d}\ell}{\mathrm{d}x}\sin(\beta - \alpha) \tag{13}$$

$$M = EI\alpha_x \tag{14}$$

The clamped–free boundary conditions can be expressed as

$$u(0, t) = v(0, t) = \alpha(0, t) = 0 \tag{15}$$

$$N(L, t) = V(L, t) = M(L, t) = 0 \tag{16}$$

Substituting $\ddot{u}$, $\ddot{v}$, and $\ddot{\alpha}$ from Eqs. (9–11) into Eq. (5), performing integration by parts, and using the boundary conditions (15) and (16), we obtain an alternative but more appealing form for Eq. (5):

$$\tau - I_h\ddot{\theta} = I_b\ddot{\theta} + \dot{H}_f = -[aV(0, t) + M(0, t)] \tag{17}$$

Note that $N(0, t) = EAu_x(0, t)$ is not accessible to the input torque τ. We further remark that the foregoing formulation also provides a general framework to accommodate beams with other constitutive behavior. Only a modification of Eqs. (12–14) is necessary.

III. Stabilization of Rest-to-Rest Maneuver

Consider the system given by Eqs. (5) and (9–16). It can be shown, with some algebra, that the equilibrium states of the system without input torque are given by $\dot{\theta}(t) = u(x, t) = v(x, t) = \alpha(x, t) = 0$ for $0 \le x \le L$ and $t \ge t_0$ (where t_0 is a real nonnegative constant). The problem is to find an appropriate control τ that drives the state of the system $(\theta, \dot{\theta}, u, \dot{u}, v, \dot{v}, \alpha, \dot{\alpha})$ from the initial state $(0, 0, \ldots, 0)$ to the target state $(\theta_d, 0, \ldots, 0)$. Motivated by Ref. 2, we write the total energy of the hub–beam system as $\varepsilon = T_{\mathrm{hub}} + T_{\mathrm{beam}} + U_{\mathrm{beam}}$, where T_{hub}, T_{beam}, and U_{beam} are the kinetic energy of the hub and kinetic and strain energies of the elastic beam, respectively. The time rate of change of ε can be computed using Eqs. (5) and (9–16), integration by parts, and tedious algebraic manipulations to yield $\dot{\varepsilon} = \tau\dot{\theta}$. This equation, however, can be obtained directly from the work–energy rate principle.[11]

Let a weighted error function with respect to the target state be the Lyapunov functional candidate

$$\phi = \varepsilon + b(T_{\text{beam}} + U_{\text{beam}}) + \tfrac{1}{2}cI_b\dot{\theta}^2 + \tfrac{1}{2}e(\theta - \theta_d)^2 \qquad (18)$$

where the real constants $b > -1$, $c > -I_h/I_b$, and $e > 0$ are design parameters. It is obvious that ϕ is positive definite and radially unbounded and has a global minimum zero at the target state. Computing $\dot{\phi}$, we obtain

$$\dot{\phi} = \dot{\theta}[\tau + b(\tau - I_h\ddot{\theta}) + cI_b\ddot{\theta} + e(\theta - \theta_d)] \qquad (19)$$

If τ is chosen taking account of Eq. (17) such that

$$\tau = (b + c)[aV(0, t) + M(0, t)] + c\dot{H}_f - k\dot{\theta} - e(\theta - \theta_d) \quad (20)$$

where the real constant $k > 0$ is also a design parameter, then the target state becomes the unique equilibrium point of the closed-loop system, and Eq. (19) reduces to $\dot{\phi} = -k\dot{\theta}^2$. We remark that the special case $c = -b$ $(-I_h/I_b < c < 1)$ corresponds to the momentum exchange feedback control proposed in Ref. 4. Note now that $\dot{\phi}$ is negative semidefinite. Thus only stability of the target state can be concluded. However, if we can show that $\dot{\theta}(t) = 0$ for $t \geq t_0$ implies $\phi(t) = 0$ for $t \geq t_0$, then globally asymptotic stability of the target state is established. Recall from Eq. (17) that $u_x(0, t)$ is not accessible to the control τ. To resolve this difficulty, we assume that either the neutral axis of the beam is inextensible in the sense $d\ell/dx = 1$ or $u_x(0, t)$ is enforced to be zero by some external agency. We proceed with the weaker condition, i.e., $u_x(0, t) = 0$, as follows.

Substituting Eq. (20) into Eq. (17) gives the closed-loop dynamics for the rigid hub:

$$(I_h + cI_b)\ddot{\theta} + k\dot{\theta} + e\theta = (1 + b)[aV(0, t) + M(0, t)] + e\theta_d \quad (21)$$

Now suppose that $\dot{\theta}(t) = 0$ for $t \geq t_0$. In view of Eq. (21), $aV(0, t) + M(0, t)$ must be a constant. Furthermore, Eqs. (5) and (17) can be combined to yield

$$aV(0, t) + M(0, t) = -\frac{d}{dt}\int_0^L \{A_\rho[(a + x + u)\dot{v} - \dot{u}v + I_\rho\dot{\alpha}]\}\, dx \qquad (22)$$

Since the closed-loop system is stable, every term in Eq. (22) is bounded. Consequently, we must have $aV(0, t) + M(0, t) = 0$ for $t \geq t_0$. It then follows from Eqs. (21) and (17) that $\theta(t) = \theta_d$ and $\tau(t) = 0$ for $t \geq t_0$. Since the remaining free vibration problem is independent of the hub radius, $aV(0, t) + M(0, t) = 0$ for $t \geq t_0$ implies $V(0, t) = M(0, t) = 0$ for $t \geq t_0$. It then leads to $v_x(0, t) = \alpha_x(0, t) = 0$ for $t \geq t_0$ by virtue of Eqs. (13–15). By assumption, we also have $u_x(0, t) = 0$ for $t \geq t_0$. From Eqs. (9–11), we obtain $u_{xx}(0, t) = v_{xx}(0, t) = \alpha_{xx}(0, t) = 0$ for $t \geq t_0$. Now it is straightforward to show, by taking repeated x differentiation of Eqs. (9–11), that all higher order partial derivatives of u, v, and α with respect to x are zero at $x = 0$. Thus we have $u(x, t) = v(x, t) = \alpha(x, t) = 0$ for $0 \leq x \leq L$, $t \geq t_0$. At this point, we have shown that $\dot{\theta}(t) = 0$ for $t \geq t_0$ implies $\phi(t) = 0$ for $t \geq t_0$. Note that the foregoing results were reached without specifying t_0. Referring now to Eq. (21), it is clear that $\dot{\theta}(t)$ approaches zero asymptotically as $t \to \infty$. Therefore, $\phi(t)$ and $\dot{\phi}(t)$ tend to zero as $t \to \infty$. Global convergence follows from Eq. (18). The proof of global asymptotic stability is completed.

IV. Conclusion

The rest-to-rest maneuver of a single-link flexible manipulator using a geometrically exact nonlinear beam model was considered. It has been shown that, with the joint torque only, a simple linear feedback law derived based on a Lyapunov-type method stabilizes the highly nonlinear coupled system described by a set of integro-partial differential equations with coupled nonlinear boundary conditions. The proof of global asymptotic stability requires the assumption of inextensibility at the root of the beam.

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Radome Slope Compensation Using Multiple-Model Kalman Filters

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Introduction

IT is well known[1–5] that if the radome shape of a missile is not hemispheric, then the incident microwave would be refracted by the radome, and there would be some pointing error. The product of radome slope and body rate would couple into the line-of-sight rate measured by the gyro on the dish antenna of the seeker. This undesirable parasitic effect would increase the miss distance and even produce low- and high-frequency instability problems for high-altitude air defense.[1] The purpose of this Note is to design a radome slope compensator based on the multiple-model Kalman filters (Ref. 6, pp. 402–407). In which the radome slope is modeled as n possible values in n sets of parallel Kalman filters, respectively. The probability of each model is obtained in a recursive manner by the Bayesian estimation scheme,[7] and the transition between each

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